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Correlation Analysis and Evolutionary Control of Multi-Staged Inverted Pendulum

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Abstract—This paper first gets the sampling values from the multi-staged inverted pendulum system controlled through the evolutionary control algorithm introduced in reference[1]. Then the strong coupling among the state variables is demonstrated using grey correlation degree analysis in grey system theory and correlation analysis in multivariable statistics. Then principal component analysis and evolutionary computation are utilized to trim redundant information and realize satisfactory control. This scheme reduces the strong coupling between different factors and has the feature of good interpretability and small computation cost. The paper finally points out the errors in one class of control algorithm.

I. Introduction

In [1], Evolutionary linear control strategy (based on the linear combination of displacement, angles and their changes) is presented to multi-staged inverted pendulums (double, triple) with the characteristics of complex non-linearity, strong duality and quick time-variance. Under the guidance of comprehensive performance function weighted ITAE, this strategy searches for the satisfactory parameters by adaptive genetic algorithm the authors presented where the crossover and mutation probability are changed with the fitness function value nonlinearly. Tracking simulations, anti-disturbance and robust experiments are given. The simulation results illustrate that this strategy has ideal dynamic, steady performance, anti-disturbance and robustness.

It is observed from the experiments that the trends of angles' variations of the first, second and third pendulums are almost identical. This strong coupling among angle variations and trends can be proved through grey correlation analysis in grey system theory and correlation analysis in multivariable statistics. Then we did PCA (Principal Component Analysis) for the system, implemented evolutionary control and obtained the simulation results. In succession, the paper points out an error in one kind of control algorithms.

II. Grey correlation analysis for state variables

Grey correlation analysis is a quantitative analysis of developmental trend of dynamic process presented by Prof. Deng Julong of HUST, P.R.China [2]. This method analytically compares geometrical shape of curves changing with time and assumes that the more similar the geometrical shape, the more close the trend, the bigger the correlation degree. Therefore, the difference between geometry shapes of curves can be used to evaluate the correlation grade. With one reference array x_0, and several series $x_1, x_2, \ldots, x_n$ to compare, the following formula is used to evaluate the gross correlation grade between the reference curve and corresponding curves to compare.

$$\xi_i(k)=\frac{\min\min|x_0(k)-x_i(k)|+\lambda\max\max|x_0(k)-x_i(k)|}{|x_0(k)-x_i(k)|+\lambda\max\max|x_0(k)-x_i(k)|} \tag{1}$$

$$r_i=\frac{1}{N}\sum_{k=1}^{N}\xi_i(k) \tag{2}$$

where ξ i(k) is the relative coefficient between the reference curve of x0 and the comparing curve of xi at kth time interval. The relative difference value in this shape is called correlation coefficient of xi to x0 at time k. λ is differentiate coefficient with value ranging from 0 to 1. It is usually set to 0.5 in our experiment. ri is defined as correlation degree between curve xi and reference curve x0.

The calculation of correlation degree is carried out in the following steps.

Step 1. Control the multi-staged inverted pendulum system using the method introduced in our recent reference[1].

Step 2. Select the data of angle variation and angle velocity of three pendulums sampled in time range that is minimally influenced by the initial state, (in this research, from the 100-th time step to the 500-th time step)

Note1: the reason no displacement and velocity are taken into account is that the authors believe the correlation between angle variations and angle velocities is most crucial. This idea is supported by the following studies

Note 2: all data are normalized before further processed – all the original data are divided by a same datum here.

Step 3. Calculate the matrix of correlation degree (TABLE I) according to equation (1), (2).

TABLE I
The matrix of correlation degree

	θ_1	θ_2	θ_3	θ'_1	θ'_2	θ'_3
θ_1	1.0000	0.4774	0.4773	0.3943	0.4792	0.4890
θ_2	0.3728	1.0000	0.8316	0.5175	0.6988	0.7435
θ_3	0.3696	0.8316	1.0000	0.5739	0.7600	0.8196
θ'_1	0.3943	0.6104	0.6593	1.0000	0.5793	0.6060
θ'_2	0.3703	0.6988	0.7600	0.4856	1.0000	0.8461
θ'_3	0.3820	0.7435	0.8196	0.5135	0.8461	1.0000

Judging from the matrix of correlation degree, the state variables are not independent. Some of them have quite big correlations, such as those between θ_2, θ_3 and between θ'_2, θ'_3.

Correlation analysis in multivariable statistics is also carried out. Correlation coefficient is an effective index to describe the degree of linear correlation of two random variables. Correlation coefficient has no scale and is not influenced by different scales used in measurement. It is a widely used index in research and application. Unlike correlation degree representing degree of geometry similarity, correlation coefficient represents linear correlation among samplings. Suppose (X_1, Y_1), (X_2, Y_2),..., (X_n, Y_n) are from a two-dimensional sampling space,

$$\rho = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}} \tag{3}$$

correlation coefficient between vector X and Y is:

We use the same data as we used to calculate correlation degree to calculate correlation coefficient (TABLE II).

TABLE II
Correlation coefficient between the variables

	θ_1	θ_2	θ_3	θ'_1	θ'_2	θ'_3
θ_1	1.0000	-0.3475	-0.1992	-0.0441	-0.0400	-0.1553
θ_2	-0.3475	1.0000	0.7921	-0.0706	-0.2491	-0.1250
θ_3	-0.1992	0.7921	1.0000	0.0721	-0.5119	-0.5590
θ'_1	-0.0441	-0.0706	0.0721	1.0000	-0.7360	-0.5299
θ'_2	-0.0400	-0.2491	-0.5119	-0.7360	1.0000	0.7027
θ'_3	-0.1553	-0.1250	-0.5590	-0.5299	0.7027	1.0000

It is obvious that there exist significant correlation coefficients among some state variables. As control variable is the weighted sum of state variables, strong correlation among state variables means information redundancy in the control process. It arises naturally that how to trim those redundant information is important. In the following, we use principal component analysis to achieve the goal.

III. Improved Evolutionary Control Algorithm

A. Principal Component Analysis

$$x'_{ij} = \frac{x_{ij} - \bar{x}_j}{s_i}, \qquad i = 1,2,3,\dots,n,\ j = 1,2,\dots,p \tag{4}$$

Principal Component analysis works on a data table with sampling points and values of state variables. It aims to extract most important information from the data. It will reduce the dimensionality of data and keep the most important, or principal component of data. The steps of principal component analysis are listed as the following[3]:

Step 1. Normalize data.

Step 2. Calculate covariance matrix $R=(x_{ij})_{n\times p}$

Step 3. Calculate the most significant m eigenvalues λ_1, $\lambda_2,\dots,\lambda_m$ ($\lambda_1 \geqslant \lambda_2 \geqslant \dots \geqslant \lambda_m$) and their corresponding eigenvectors u_1, u_2, ..., u_m. Those eigenvectors are orthogonal and called principal axis.

Step 4. $e_i = (e_i(1)\ \ e_i(2)\ \ \dots\ e_n(p))'$ is i-th sample. As e_i is centralized, then the h-th principal component is:

$Y_h=(y_h(1)\ \ y_h(2)\ \ \dots\ \ y_h(n))'$

$Y_h(j)=e'_j u_h$, $j=1,2,\dots,n$

Therefore:

$Y_h= (e_1\ \ e_2\ \ \dots e_n)' u_h=\Sigma u_h(j) x_i$

Thus, Y_h is linear combination of original state variables x_i (i=1,2, ..., p) and coefficients of the combination is exactly:

$U_h=(u_h(1)\ \ u_h(2)\ \ \dots\ \ u_h(n))'$ $\qquad h=1,\dots,m$

B. Principal Component Regression

In a d-dimensional space, d components can be obtained. In applications, the most significant several principal components are used to do analysis. Selection of the most significant P principal components as the elements to calculate and optimize control variable leads to a new control scheme. Here:

$$PI\% = \sum_{i=1}^{p}\lambda_i \Big/ \sum_{i=1}^{d}\lambda_i \tag{5}$$

Where PI means preserved information.

C. Evolutionary Control of Inverted Pendulum Based on Principal Component Analysis

Data was first obtained through certain methods, the data in the time interval that control is most stable are taken to do analysis. Here we only consider angles and angular velocities of three pendulums because displacements and displacement velocities are only weakly related with them.

Two kinds of data processing approaches are used in the experiment. 1) Normalize data according to equation (4); 2) Omit the step of data normalization and calculate covariance matrix R directly. The reason approach 2) is prefered in this paper is because all the data is selected approximate to the equilibrium point while large change may occur because of small variance by approach 1). Data obtained from approach 2) is reported as the following(TABLEIII):

TABLE III
Covariance of the variables

	θ_1	θ_2	θ_3	θ'_1	θ'_2	θ'_3
θ_1	0.1256	-0.0059	-0.0026	-0.0069	-0.0037	-0.0098
θ_2	-0.0059	0.0023	0.0014	-0.0015	-0.0031	-0.0011
θ_3	-0.0026	0.0014	0.0013	0.0012	-0.0049	-0.0036
θ'_1	-0.0069	-0.0015	0.0012	0.1936	-0.0849	-0.0417
θ'_2	-0.0037	-0.0031	-0.0049	-0.0849	0.0688	0.0330
θ'_3	-0.0098	-0.0011	-0.0036	-0.0417	0.0330	0.0320

Eigenvalues and eigenvectors are calculated as :
λ_1=0.2492, λ_2=0.1275, λ_3=0.0333, λ_4=0.0117,
λ_5=0.0001, λ_6=0.0017
U_1=[0.0163 -0.0020 -0.0164 -0.8624 0.4488 0.2331]';
U_2=[-0.9909 0.0437 0.0157 0.0401 0.0597 0.1041]';
U_3=[0.1067 -0.1336 -0.1564 0.4880 0.6498 0.5347]';
U_4=[-0.0486 -0.0841 0.0148 0.0879 0.5861 -0.7994]';
U_5=[-0.0066 0.4453 -0.8901 -0.0241 -0.0309 -0.0883]';
U_6=[-0.0641 -0.8803 -0.4271 -0.0907 -0.1682 -0.0448]'

Select the eigenvectors of the first four eigenvalues to get a new form of control scheme and a set of parameters to be optimized. Evolutionary computation is used to optimize the parameters. Simulation results are shown below:

(1) results without noise (Fig.1)

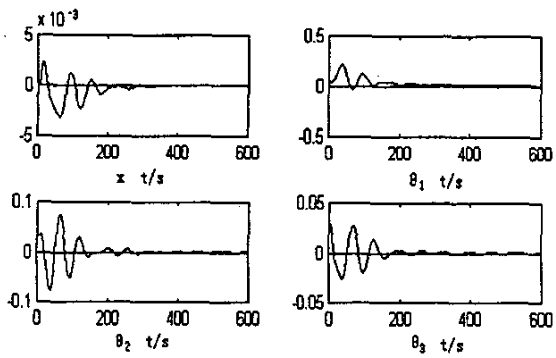

Fig. 1. The Simulated Curves without Noise

ITAE=0.402 ;
K=[44.3725 , -46.2113 , 3.29143 , -13.958 , 0.407163 , 15.8039]

(2) results of robustness (Fig.2)

Initial states are: θ_1=-0.03rad , θ_2=0.03rad , θ_3=0.03rad (*K* is as the above)

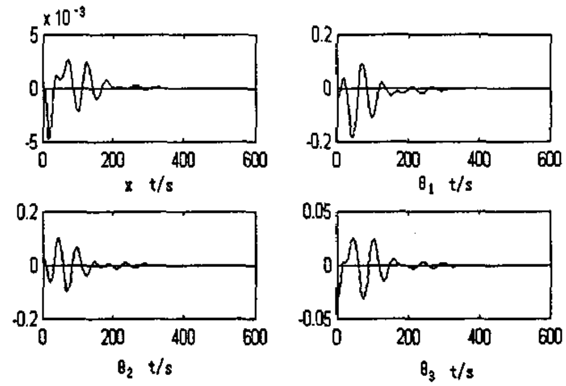

Fig. 2. The Results of Robustness

(3) results when random noise is added to (Fig.3)

Where x is added noise $n \in$ [-0.001m,0.001m]; θ_1 is added noise $n \in$ [-0.015rad,0.015rad].

We also carry out experiments by the approach 1), the effects are well though approach 2) are more better. Experiment results demonstrate good static and dynamic characteristics, insensitivity to interferences, and robustness of evolutionary control scheme based on principal component analysis for multi staged inverted pendulum system. Compared to the scheme in [1], this scheme reduces the strong coupling between different factors and has the feature of good interpretability and smaller computation cost during the optimizing process.

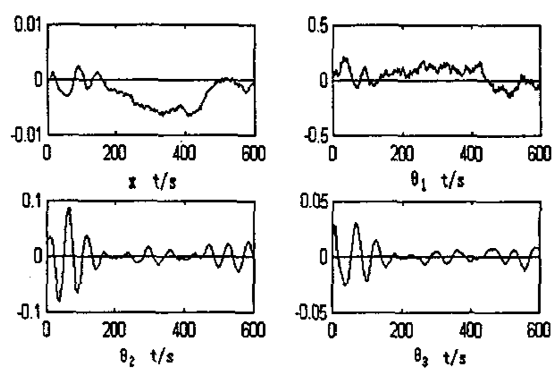

Fig. 3. The Curves of Disturbance-resistance

IV. Comment on a kind of control strategy

Many researchers used one class of control scheme for inverted pendulum as the following [4-6]:

For the first inverted pendulum:

$$U = k_1 x + k_2 \dot{x} + k_3 \theta_1 + k_4 \dot{\theta}_1 \tag{7}$$

For the second inverted pendulum:

$$U = k_1 x + k_2 \dot{x} + k_3 \theta_1 + k_4 \dot{\theta}_1 - k_5 \theta_2 - k_6 \dot{\theta}_2 \tag{8}$$

For the third inverted pendulum:

$$U = k_1 x + k_2 \dot{x} + k_3 \theta_1 + k_4 \dot{\theta}_1 - k_5 \theta_2 - k_6 \dot{\theta}_2 + k_7 \theta_3 + k_8 \dot{\theta}_3 \tag{9}$$

where k_i (i=1,...,8) are all positive. Obviously, for the adjacent stage pendulums, the symbols are different.

The above control strategy does not consider the strong coupling of state variables in specifying the negative and positive of coefficients. As the influences of movements of one pendulum upon others are very complicated due to the strong correlation of different ones, it is not advisable to predefine the polarity of coefficients of state variables. Therefore the authors suggest utilization of principal component analysis, or carrying out the optimizing procedure to get the optimal coefficients k_i (i =3,...,8) in a symmetric interval.

V. Conclusions

This paper first optimizes the parameters for a linear controller using genetic algorithm, then proves the strong correlation among state variables (angle) in control scheme[1] using grey correlation degree analysis and correlation analysis. To get rid of the redundant information in control strategy, principal component analysis is integrated into genetic algorithm to realize satisfactory control. Results of the simulation experiments are promising. The paper also discusses problems of one kind of control scheme and presents suggestions for correction.

Though promising results have been achieved using our control scheme, there still exist some setbacks to be overcome:

(1) With a high computational cost, it is difficult to realize online optimization and control.

(2) There is a lack of guidelines on how to select data set properly in implementing principal component analysis.

In addition, we only investigated inverted pendulum system with horizontal cart. Our next plan is to further study the control of inverted pendulum system in skewed cart and develop a more robust intelligent controller that can adapt to the changing environments online.

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